

THE MATHEMATICAL ANALYSIS OF ULTRASONIC WAVES PROPAGATION APPLICATED TO REALIZATION OF HIGH POWER PIEZOELECTRIC TRANSDUCERS

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Abstract. *The high energy ultrasound is generated by piezoelectric elements which are excited from electric signal obtained from an electronic generator. The transmission of this high energy is obtained by transmission or concentrator elements, in function of what we want to do with this energy utilized especial in no conventional technologies. The efficiency work is very important aspect because the energy utilizes has high values. In this paper is presented methods to calculate the elements components necessary to generate and transmits this high ultrasound energy: piezoelectric transducers and transmission/concentrator elements. Is presented the experimental results obtained with the theory presented. Original contribution consists by method used and suggestive graphics for appreciation of parameters variations.*

Keywords: *piezoelectric transducer, ultrasound power, dimension design.*

Mathematics Subject Classification 2010: 74J05

1. INTRODUCTION

In paper [1] is described a few methods to increase the efficiencies of acoustic systems especial by point of view of electronic generator and its adaptation with acoustic system. In papers [2] and [3] is taken in account a various types of adaptation schemes between electronic generator and acoustic components. In paper [4] is presented a method to design a few resonances transformation/adaptation bodies in function of work conditions and acoustic charge. In paper [5] is presented a few applications of high ultra acoustic energy utilized in fabrication of soap.

In this paper is utilizes the general principles described in [2], [3] and [4] for to design and characterization the piezoelectric transducers. Is find out a work procedure to calculate the physical dimensions of element which constitutes the piezoelectric transducer in function of composition material, work parameters and analyzing there influences on characteristic parameters of piezoelectric transducer.

In practice, [4], we meet two important cases in propagation of ultrasonic waves. One of these is the case of the transmission/concentrator elements, when the ultrasound waves pass through one propagation medium (usual aluminum alloy or titan) and the second is the case of propagation of ultrasound waves through three mediums (usual steel - piezoelectric element-aluminum alloy). This is the case of ultrasound piezoelectric transducer.

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2. THE PRINCIPLE OF OPERATION

In case of a piezoelectric transducer, [2], formed by two pastilles, which are catch between two metallic blocks (named director and reflector) by a central screw. The used materials in reflector composition are: steel, aluminum alloy, bronze, naval brass. The used materials in director composition are: titanium, aluminum alloy or magnesium composite.

The long of transducer, by resonance conditions, is $\lambda/2$ – Fig.1 (λ represents the wave long of ultrasound wave through material). Is noted:

$$K_i = \frac{2\pi}{\lambda_i} = \frac{2 \cdot \pi \cdot f}{v_i} = \frac{\omega}{v_i} \quad (1)$$

$$K_C = \frac{2\pi}{\lambda_C} = \frac{2 \cdot \pi \cdot f}{v_C} = \frac{\omega}{v_C}$$

where:

- K_i, K_C are constants which referrers to the end piece respectively to the ceramic material.
- λ_i and λ_C represents the waves long of ultrasound wave through end material and ceramic material, [m];
- v_i and v_C represents propagation speed through end material and ceramic material, [m/s];
- f represents frequency of excited wave [Hz] and $\omega=2\pi f$ represents its pulsation.

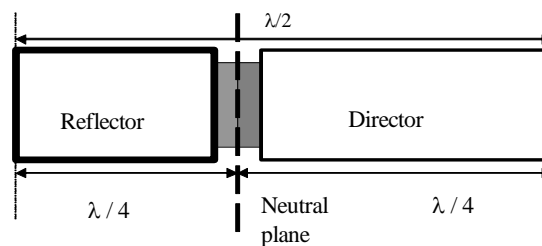


Fig. 1 The piezoelectric transducer.

The propagation speed through material, noted v , depends by:

ρ - the material density, [Kg/m^3];

Y - the elasticity module, [N/m^2];

σ - the Poisson coefficient.

Is considered a piezoelectric transducer presented in Fig. 2, where, below of transducer, is represented, [3], the evolution of vibration amplitude $y(x)$ in function of distance x .

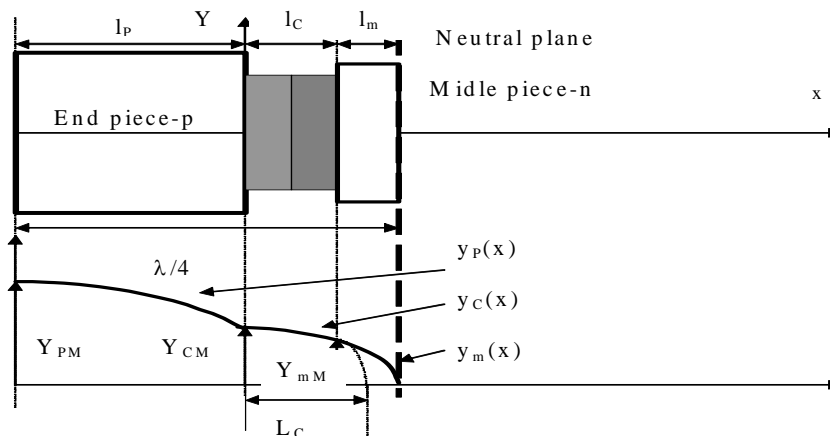


Fig. 2. Semi transducer with three propagation mediums.

$$2) y_C(l_C) = y_m(l_C) \text{ or } Y_{CM} \sin[K_C(x - L_C)] = Y_{mM} \sin[K_m(x - l_C - l_m)] \Rightarrow$$

$$\frac{Y_{CM}}{Y_{mM}} = \frac{\sin K_m \cdot l_m}{\sin[K_C \cdot (l_P - L_C)]}$$

It puts the limit conditions for forces - b)/3 conditions - and is obtained:

$$3) F_P(0) = F_C(0). \quad (5)$$

It is obtained:

$$-Y_P \cdot A_P \cdot Y_{PM} \cdot K_P \cdot \sin(K_P l_P) = Y_C \cdot A_C \cdot Y_{CM} \cdot K_C \cdot \cos(-K_C L_C)$$

$$-\frac{Y_{PM} \cdot \sin K_P l_P}{Y_{CM} \cdot \cos K_C L_C} = \frac{Y_C \cdot A_C \cdot K_C}{Y_P \cdot A_P \cdot K_P} \quad (6)$$

Replacing the ratio $\frac{Y_{PM}}{Y_{CM}}$ with the value finds out from a)/1 condition, relation (4):

$$-\left(-\frac{\sin K_C l_C}{\cos K_P l_P}\right) \cdot \frac{\sin K_P l_P}{\cos K_C L_C} = \frac{Y_C \cdot A_C \cdot K_C}{Y_P \cdot A_P \cdot K_P} \quad (7)$$

Taken $l_C \approx L_C$ and is obtained:

$$\operatorname{tg} K_P l_P \cdot \operatorname{tg} K_C L_C = \frac{Y_C \cdot A_C \cdot K_C}{Y_P \cdot A_P \cdot K_P} = \frac{\rho_C \cdot v_C \cdot A_C}{\rho_P \cdot v_P \cdot A_P} = \frac{Z_C \cdot A_C}{Z_P \cdot A_P} = R_{CP} \quad (8)$$

Because we have, from definition: $v = \sqrt{\frac{Y}{\rho}}$ (for bars) and $K_i = \frac{\omega}{v_i} = \frac{2\pi}{\lambda_i}$ ($v_i = f \cdot \lambda_i$).

In relation (8) $Z_C = \rho_C \cdot v_C$ and $Z_P = \rho_P \cdot v_P$ represents the acoustic impedances for ceramics and for the end piece.

It puts the second condition, b)/4 for forces and obtained relations (9) and (10):

$$4). F_C(l_C) = F_m(l_C). \quad (9)$$

$$Y_C \cdot A_C \cdot Y_{CM} \cdot K_C \cdot \cos[K_C(l_C - L_C)] = Y_m \cdot A_m \cdot Y_{mM} \cdot K_m \cdot \cos[K_m(-l_m)]$$

$$\frac{Y_{CM}}{Y_{mM}} \cdot \frac{\cos[K_C(l_C - L_C)]}{\cos K_m l_m} = \frac{Y_m \cdot A_m \cdot K_m}{Y_C \cdot A_C \cdot K_C} \quad (10)$$

Replacing the ratio $\frac{Y_{CM}}{Y_{mM}}$ with the value finds out from a)/2 condition, relation (4):

$$\frac{(-) \cdot \sin K_m l_m}{\sin[K_C(l_C - L_C)]} \cdot \frac{\cos[K_C(l_C - L_C)]}{\cos K_m l_m} = \frac{Y_m \cdot A_m \cdot K_m}{Y_C \cdot A_C \cdot K_C} \quad (11)$$

$$- \operatorname{tg} K_m l_m \cdot \operatorname{ctg} [K_C (l_C - L_C)] = \frac{Y_m \cdot A_m \cdot K_m}{Y_C \cdot A_C \cdot K_C} = R_{mC}$$

In conclusion we have obtained:

$$\begin{aligned} \operatorname{tg} K_C L_C \cdot \operatorname{tg} K_P l_P &= R_{CP} \\ - \operatorname{tg} K_m l_m \cdot \operatorname{ctg} [K_C (l_C - L_C)] &= R_{mC} \end{aligned} \tag{12}$$

where:

$$\begin{aligned} R_{CP} &= \frac{Y_C \cdot K_C \cdot S_C}{Y_P \cdot K_P \cdot S_P} = \frac{\rho_C \cdot v_C \cdot S_C}{\rho_P \cdot v_P \cdot S_P} = \frac{Z_C \cdot S_C}{Z_P \cdot S_P}; \\ R_{mC} &= \frac{Y_m \cdot K_m \cdot S_m}{Y_C \cdot K_C \cdot S_C} = \frac{\rho_m \cdot v_m \cdot S_m}{\rho_C \cdot v_C \cdot S_C} = \frac{Z_m \cdot S_m}{Z_C \cdot S_C} \end{aligned} \tag{13}$$

It notes: $K_C L_C = \alpha_L$; $K_P l_P = \alpha_P$; $K_m l_m = \alpha_m$; $K_C l_C = \alpha_C$
 The equation (12) becomes:

$$\operatorname{tg} \alpha_L \cdot \operatorname{tg} \alpha_P = R_{CP} \quad \text{and} \quad \operatorname{tg} \alpha_m \cdot \operatorname{ctg} (\alpha_C - \alpha_L) = -R_{mC} \tag{14}$$

Is obtained:

$$\begin{aligned} \operatorname{tg} \alpha_L \cdot \operatorname{tg} \alpha_P = R_{CP} &\Rightarrow \operatorname{tg} \alpha_L = \frac{R_{CP}}{\operatorname{tg} \alpha_P}; \\ \operatorname{tg} \alpha_m \cdot \frac{1 + \operatorname{tg} \alpha_C \cdot \operatorname{tg} \alpha_L}{\operatorname{tg} \alpha_C - \operatorname{tg} \alpha_L} &= -R_{mC} \end{aligned} \tag{15}$$

Replacing $\operatorname{tg} \alpha_L$ in the second equation [15], is obtained:

$$\operatorname{tg} \alpha_m \cdot \frac{1 + \operatorname{tg} \alpha_C \cdot \frac{R_{CP}}{\operatorname{tg} \alpha_P}}{\operatorname{tg} \alpha_C - \frac{R_{CP}}{\operatorname{tg} \alpha_P}} = -R_{mC} \tag{16}$$

Or relation:

$$\operatorname{tg} \alpha_m \cdot \operatorname{tg} \alpha_P + R_{CP} \cdot \operatorname{tg} \alpha_m \cdot \operatorname{tg} \alpha_C + R_{mC} \cdot \operatorname{tg} \alpha_C \cdot \operatorname{tg} \alpha_P = R_{mC} \cdot R_{CP} \tag{17}$$

From R_{mC} si R_{CP} expressions is obtained:

$$R_{mC} \cdot R_{CP} = \frac{\rho_m \cdot v_m \cdot S_m}{\rho_C \cdot v_C \cdot S_C} \cdot \frac{\rho_C \cdot v_C \cdot S_C}{\rho_P \cdot v_P \cdot S_P} = \frac{\rho_m \cdot v_m \cdot S_m}{\rho_P \cdot v_P \cdot S_P} = R_{mP} \tag{18}$$

This relation is noted with R_{mP} and shows the transition from metal (m-director) to end piece (p-reflector) – fig. 1. This relation permits to calculate the dimensions of semi-

transducer having 3 propagation mediums [4]. It starts from neutral plane which is determined (l_m is determined). The determination of l_m , l_C or l_P dimensions when other sizes are know, may be effected from one of relations given down, which are obtained from the same equation (17):

$$\begin{aligned} l_m &= \frac{\alpha_m}{K_m} = \frac{1}{K_m} \cdot \arctg R_{mC} \cdot \frac{R_{CP} - \tg\alpha_C \cdot \tg\alpha_P}{\tg\alpha_P + R_{CP} \cdot \tg\alpha_C} \\ l_P &= \frac{\alpha_P}{K_P} = \frac{1}{K_P} \cdot \arctg R_{CP} \cdot \frac{R_{mC} - \tg\alpha_m \cdot \tg\alpha_C}{\tg\alpha_m + R_{mC} \cdot \tg\alpha_C} \\ l_C &= \frac{\alpha_C}{K_C} = \frac{1}{K_C} \cdot \arctg \cdot \frac{R_{mP} - \tg\alpha_m \cdot \tg\alpha_P}{R_{CP} \cdot \tg\alpha_m + R_{mC} \cdot \tg\alpha_P} \end{aligned} \quad (19)$$

where, l_C represents the total thickness of whole package ceramics of semi-transducer considered.

Because ceramics isn't placed into nodal plane (where we have the null displacements $y=0$ and maximum mechanical tensions $T=\max$) for to obtain the same piezoelectric effect it shall to increase the thickness of piezoelectric elements package. This is disadvantageous by economic point of view. Is preferred to placed nodal plane in proximity of piezoelectric elements but situates in director piece where it realized the cached of assembly transducer, because the displacements is null in this zone.

To calculate the transversal dimensions, necessary for a given level of power, we take in account two parameters: the maximum stress and dynamic deformations admissible for materials from which it is constitute the piezoelectric transducer.

The stress (T), respectively dynamic deformations ($S=\Delta l/l$) into a resonant mechanical element are in function of permissible displacement at end element. This displacement is determined by acoustic charge. Let's find out a relation between displacement at end element and maximum tension which take place at neutral plane from the transducer.

Let's note:

- Y_0 - the maximum amplitude displacement, situated at radiation face of transducer;
- y_m - the amplitude displacement at x distance from nodal plane.

The nodal plane represents the section of transducer where the displacements are nulls and mechanical tensions are maximums.

It can writes – fig. 3:

$$y_m(x) = Y_0 \cdot \sin \omega \cdot x = Y_0 \sin \frac{2\pi}{\lambda} \cdot x \quad (20)$$

The value of relative deformation S for any x value:

$$S(x) = \lim_{\Delta x \rightarrow 0} \frac{y_m(x + \Delta x) - y_m(x)}{\Delta x} = \frac{dy_m(x)}{dx} \quad (21)$$

Results:

$$S(x) = Y_0 \frac{2\pi}{\lambda} \cdot \cos \frac{2\pi}{\lambda} \cdot x \quad (22)$$

The maximum relative deformation $S(x)$ in nodal plane ($x=0$):

$$S_{\max}(x=0) = Y_0 \cdot \frac{2\pi}{\lambda} = Y_0 \cdot \frac{2\pi}{v} \cdot f = \frac{\omega \cdot Y_0}{v} \tag{23}$$

where: v – represents the speed sound through material, [m/s];

$\omega=2\pi f$ – vibrations pulsation, [Hz];

$\lambda=v/f$ – the length wave of vibrations through material, [m].

The maximum amplitude of dynamic vibrations T_m (or the maximum amplitude of dynamic pressure P_m) will give by Hooke law:

$$T_m = Y \cdot S_m = \frac{F}{A},$$

where: Y - represents the elasticity module of material, [N/m²];

F - represents the force, [N];

A - represents the section area, [m²].

Also, we have:

$$v = \sqrt{\frac{Y}{\rho}}$$

in the case of bars (having the same diameter from it along).

where; v - represents the sound speed through material, [m/s];

ρ - represents, the material density of transducer pieces, [kg/m³].

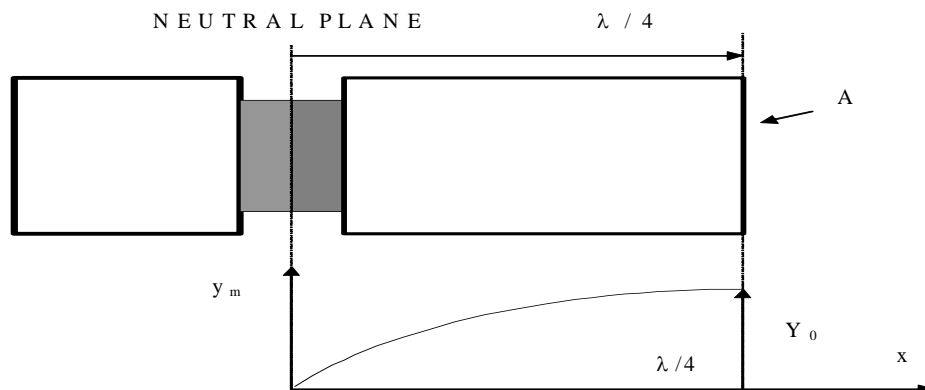


Fig. 3. Transversal dimensioning.

The maximum amplitude of vibration speed (noted v_m) at end of element is given by relation:

$$y_m(x) = Y_0 \sin \omega x \Rightarrow v_m(x) = \frac{dy_m(x)}{dx} = Y_0 \cdot \omega \cdot \cos \omega x$$

Results, with condition $\cos \omega x = 1$ and utilizing relation (23) for $\omega \cdot Y_0$:

$$v_m = \omega \cdot Y_0 = v \cdot S_{\max} = v \cdot \frac{T_m}{Y} = v \cdot \frac{T_m}{v^2 \cdot \rho} = \frac{T_m}{\rho \cdot v} \quad \text{or: } T_m = P_m = \rho \cdot v \cdot \omega \cdot Y_0$$

So, the mechanical tension (T_m) or acoustic pressure (P_m) at nodal plane is given by product between maximum speed of particles (v_m) at this end and specific acoustic impedance

$$(Z = \rho \cdot v): T_m = P_m = \rho \cdot v \cdot v_m = Z \cdot v_m.$$

The acoustic intensity I , represents the acoustic energy flux which pass through unit surface perpendicular on propagation direction of waves. It is given by relation:

$I = \frac{1}{2} \cdot v_m \cdot P_m$ where: v_m – represents the maximum speed amplitude in middle piece-director, at end face, [m/s];

P_m - represents the pressure amplitude in middle piece-director, [N/m²].

By overtaking value for I_{\max} it may destroy the transducer via:

- by overtaking value for mechanical tensions T_m [N/m²], usual [daN/cm²] or
- by overtaking value for temperature T [°C].

We have the relations:

$$v_m = \frac{T_m}{\rho \cdot v} = \frac{P_m}{\rho \cdot v} \quad ; \quad Z = \rho \cdot v \quad (24)$$

Replacing, results:

$$\begin{aligned} I &= \frac{1}{2} \cdot \frac{P_m}{\rho \cdot v} \cdot P_m = \frac{1}{2} \cdot \frac{P_m^2}{\rho \cdot v} && \text{(expressed by } P_m\text{);} \\ I &= \frac{1}{2} \cdot v_m \cdot v_m \cdot \rho \cdot v = \frac{1}{2} \cdot \rho \cdot v \cdot v_m^2 && \text{(expressed by } v_m\text{)} \end{aligned} \quad (25)$$

Is noted: $Z_1 = \rho_1 v_1$ - the transducer impedance; $Z_2 = \rho_2 v_2$ - the medium impedance where it is transmit the energy flux.

At separation limit transducer/medium these two acoustic intensities are equals: $I_1=I_2=I$. Replacing in relation (25) relation (24), will obtain:

$$I = \frac{1}{2} \cdot \rho_2 \cdot v_2 \cdot v_m^2 = \frac{1}{2} \cdot \rho_2 \cdot v_2 \cdot \left(\frac{T_m}{\rho_1 \cdot v_1} \right)^2 \quad (26)$$

The relation is valid in case of a homogeneous transducer. In case of a composite transducer, we have an amplification coefficient G_1 given by relation:

$$I = \frac{1}{2} \rho_2 v_2 v_m^2 G_1 = \frac{1}{2} \rho_2 v_2 \left(\frac{T_m}{\rho_1 \cdot v_1} \right)^2 G_1 \quad (27)$$

The debited power by transducer through end surface A will be:

$$P = I \cdot A = \frac{1}{2} \cdot \rho_2 \cdot v_2 \cdot \left(\frac{T_m}{\rho_1 \cdot v_1} \right)^2 \cdot G_1 \cdot A \quad (28)$$

This is the power in function of material parameters. For it calculates we proceed:

- It imposes $\rightarrow T_m$ - which not pass the admissible maximum tensions (by point of view of fatigue);

$\rightarrow P$ - debited power.

- It knows $\rightarrow \rho_1 v_1$ – the end of transducer (director);

$\rightarrow \rho_2 v_2$ – the work medium;

→ G_1 - the amplification.

Results A – the transversal area of transducer necessary to generated the impose power.

The power which may be debits from transducer it is depending from acoustic impedance charge $Z_2 = \rho_2 \cdot v_2$.

For a small acoustic $Z_2 \rightarrow 0$, the director will vibrate with a large amplitude and so not can appliqué high power by point of view of safety. If we have: water like acoustic charge → $Z_2 = \rho_2 v_2 = 1,5 \cdot 10^6 \text{ Kg / m}^2\text{s}$; and air like acoustic charge → $Z_2 = \rho_2 v_2 = 420 \text{ Kg / m}^2\text{s}$.

Therefore, the admissible power, in case of air, is much small. The transversal dimensions aren't being too big. It is impose to be $\leq \frac{1}{2}$ from longitudinal dimension of transducer ($\lambda/2$) for not permit to appear the transversal resonance belong of piezoelectric transducer.

3. EXPERIMENTAL RESULTS

A piezoelectric transducer, having 500W power at 20 kHz work frequency is given in Fig. 4. In these situations is preferable to use the velocity transformations for to magnify the vibration amplitude. These velocity transformations are composed from aluminum alloy resonators having a long given by $\lambda/2$ ($\lambda = 256 \text{ mm}$ in case of a 20 kHz frequency) - Fig. 5.

Is built two acoustic systems and with 1000Vvv excitation signal is obtained:

- a. The acoustic system I, having $D_2 = 52 \text{ mm}$; $D_3 = 25 \text{ mm}$ diameters. The vibration amplitude was $50 \mu\text{m}$.
- b. The acoustic system II, having $D_2 = 59 \text{ mm}$; $D_3 = 22 \text{ mm}$ diameters. The vibration amplitude was $80 \mu\text{m}$.

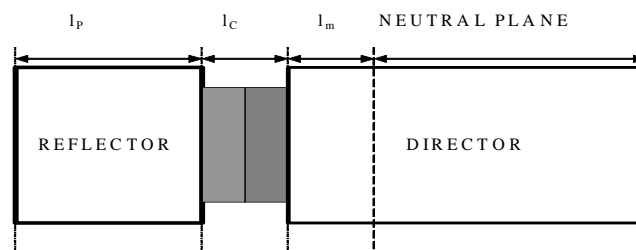


Fig. 4. 500W / 20 kHz transducer.

In both systems it is utilizes an acoustic transformation having $D_4 = 60 \text{ mm}$ and $D_5 = 26 \text{ mm}$ diameters. The amplification obtained with these two acoustic systems is given by:

$$G = \left(\frac{D_2}{D_3} \cdot \frac{D_4}{D_5} \right)^2 \quad \text{in conformities with general formulas} \quad G = \left(\frac{D_{\max}}{D_{\min}} \right)^2 . \text{ Replacing is obtained:}$$

- The acoustic system I ⇒ $G_1 = 23$;
- The acoustic system II ⇒ $G_2 = 39,3$

These coefficients will represent the amplifications realized from these acoustic impedances in these two cases. In both cases we have an amplification of acoustic impedance and an amplification of transducer charge.

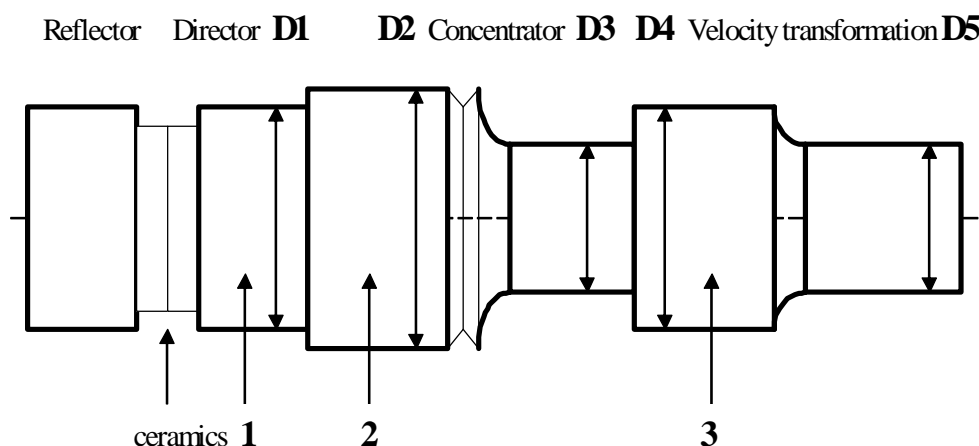


Fig. 5. Acoustic chain-transducer (1), concentrator (2), velocity-transformation (3).

4. CONCLUSIONS

With help of the relations presented in paper we can calculate the dimensions of a piezoelectric transducer used to produce the power ultrasonic field necessary at unconventional technology [5] like washes, solders etc. The results were verified in practice and help us to quickly find the dimensions and the influence of dimensions on global performances.

With material and financial support given by S.C.Tehnofina S.A. and used theory presented is realized and homologated a few planar transducers in power range (50-250)W whose may be used at washing and miscellaneous ultrasonic technology. It was realized and homologated, also, a piezoelectric transducer having 1000W power, which may be utilized at washing and solders based by ultrasonic field.

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